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Vibroacoustic Modeling for Space Shuttle Orbiter Thermal Protection System

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A ceramic tile has been developed as part of the thermal protection system of the Space Shuttle orbiter. These tiles are individually attached to the panels of the orbiter through a flexible pad. Closed-form and numerical linear random vibration models were formulated to predict tile dynamic stresses due to the broadband acoustic field on the outer surface and the base excitation on the bonded surface. Analytical results were compared to a full-scale random test. The analytical models were then used to predict dynamic stresses on selected tiles. Conditions were established as to the accuracy of the closed-form techniques.

Nomenclature

A_s = strain isolation pad (SIP) surface area
A_t = tile exposed surface area
c = linear viscous damping coefficient of SIP
dB = decibels
dω = frequency differential, rad/s
f_i = ith frequency, Hz
f_n = natural frequency of tile, Hz
$F(t)$ = acoustic force acting on tile
g = acceleration of gravity
$H(f_i)$ = transfer function between tile acceleration and panal acceleration for the ith one-third octave band
$H_1(f_i)$ = transfer function between tile velocity and acoustic pressure for the ith one-third octave band
$H_2(f_i)$ = transfer function between tile displacement and acoustic pressure for the ith one-third octave band
$H(\omega)$ = linear transfer function between tile acceleration and panel acceleration
$H_1(\omega)$ = linear transfer function between tile velocity and acoustic pressure
$H_2(\omega)$ = linear transfer function between tile displacement and acoustic pressure
k = linear stiffness of SIP
m = mass of tile
PSD = power spectral density
P_a = three-sigma stress in SIP due to acoustic field
P_c = viscous damping component of three-sigma stress in SIP due to acoustic field
P_d = three-sigma stress in SIP due to panel vibration
P_k = stiffness component of three-sigma stress in SIP due to acoustic field
P_t = total three-sigma stress in SIP due to panel vibration and acoustic field
r = ratio of forcing frequency and natural frequency
SIP = strain isolation pad
$S_p(f_i)$ = acoustic pressure PSD at the ith one-third octave band
$S_p(\omega)$ = acoustic pressure PSD
$\bar{S}_p$ = maximum acoustic pressure PSD
$S_{\dot{x}}(\omega)$ = tile velocity PSD
$S_{\ddot{x}}(\omega)$ = tile acceleration PSD
$S_{\ddot{y}}(f_i)$ = panel acceleration PSD at the ith one-third octave band
$S_{\ddot{y}}(\omega)$ = panel acceleration PSD
$\bar{S}_{\ddot{y}}$ = maximum panel acceleration PSD
TPS = thermal protection system
W = weight of tile
$\overline{X^2}$ = mean square displacement of tile
$\overline{\dot{X}^2}$ = mean square velocity of tile
$\overline{\ddot{X}^2}$ = mean square acceleration of tile
y = panel displacement
Δf_i = bandwidth of the ith one-third octave band, Hz
ζ = damping ratio of tile/SIP

Introduction

ONE major engineering innovation associated with the Space Shuttle program has been the design, development, and installation of sintered silicate fiber tiles flexibly attached to the orbiter's aluminum skin, Fig. 1. Each tile is bonded to an orbiter panel through a strain isolation pad (SIP). The SIP is a Nomex felt material consisting of two layers of interwoven fibers stitched together. The thickness of the SIP is typically 0.16 in. (4.1 mm). The tiles, of which there are more than 30,000, are designed to be a thermal barrier to protect the outer aluminum surface panels of the orbiter from the high temperatures during reentry.

There are two structurally critical mission phases for which the tiles must be designed. These phases are at liftoff and during passage through the high dynamic pressure region of ascent. At these times the vehicle is exposed to high-level broadband random pressure fluctuations that cause the orbiter's outer panels to vibrate. These random vibrations produce a base excitation load on the tiles. The actual dynamic response is a coupled motion between the tiles and panels with the acoustic field acting as the forcing function.

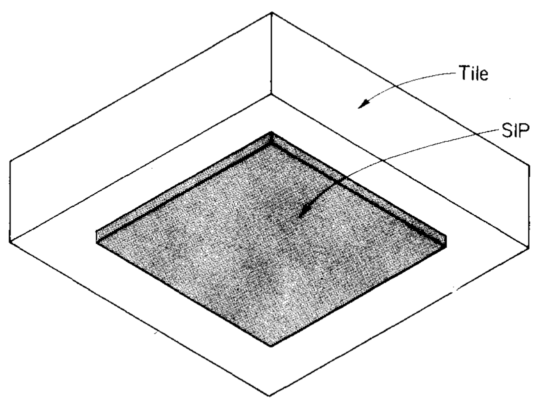

Fig. 1 Space Shuttle orbiter thermal protection system.

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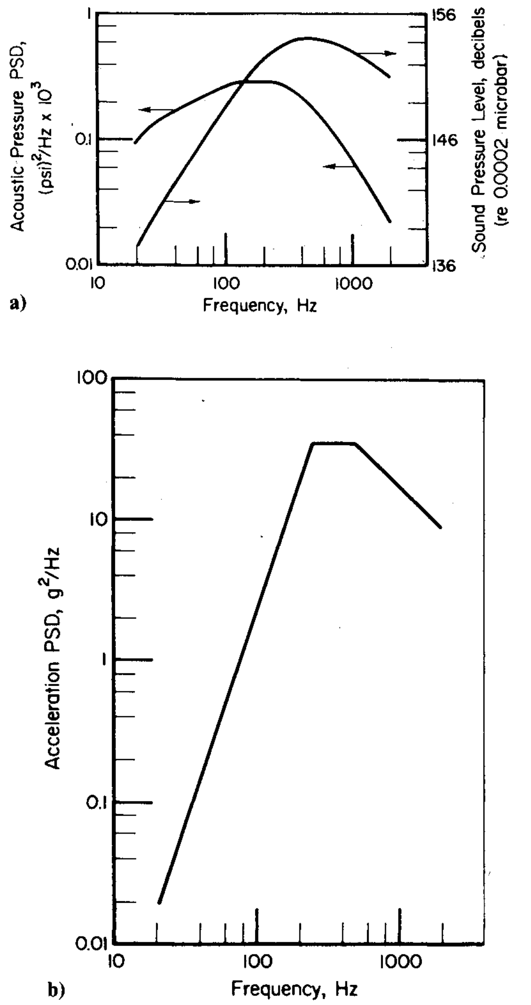

Fig. 2 Vibroacoustic environment of orbiter body flap: a) acoustic field,[1] b) panel vibration.[2]

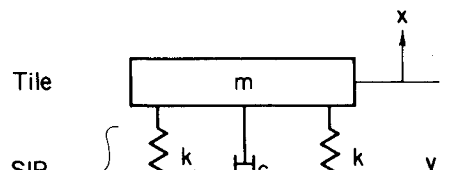

Fig. 3 Base excitation of a linear mass/spring/damper system.

This paper presents the author's formulation of linear analytical models used to predict loads at the SIP/tile interface. It also compares analytical results with those of a full-scale acoustic test on the body flap. Finally, vibroacoustic loads are predicted for selected tiles using the techniques developed in this paper.

Mission Environment

The acoustic fields surrounding the orbiter during liftoff and ascent have been established by small-scale tests and by comparison with full-scale Saturn V liftoffs.[1] The pressures were determined at specific locations, and then generalized to regions of the orbiter's outer boundaries. The acoustic field for each region was tabulated as a sound pressure level for one-third octave bands from 20 to 2000 Hz. Figure 2a shows the maximum acoustic pressure at liftoff for the body flap.

The orbiter panel vibrations were given in terms of acceleration power spectral densities (PSD's) for the worst location in a region of the orbiter's surface.[2] The acceleration PSD's were generally specified from 20 to 2000 Hz, Fig. 2b. Generally, the acceleration PSD specifications were characterized by a constant value over several hundred hertz, and a three, six, or nine dB/octave rolloff on either side of the constant level. Both PSD's shown in Fig. 2 are envelopes of many test results. They therefore represent conservative estimates of the total random environment that would be experienced by a tile. They do not include the effect of shocks.

Mathematical Models

Assumptions

To develop a reasonably simple method of analysis for the tile's random vibrations, it was necessary to consider a number of simplifying assumptions. The validity of the analysis resulting from these assumptions can be demonstrated by more detailed analysis techniques or, ideally, by testing.

Each tile was considered as an independent element subjected to the random vibrations of an orbiter surface panel and the pressure fluctuations of the acoustic field. Although the panel vibrations are caused by the acoustic field, the two phenomena have been analyzed separately and combined to give a total vibroacoustic loading.

Since both forcing functions were wide-band excitations in the frequency domain, a random vibrations approach was necessary to predict loadings. In so doing, a mean square response was determined and then converted to a three-sigma loading. Based on the conservative input excitations, Fig. 2, a three-sigma design load is a reasonable upper bound for design, and was so specified for the Space Shuttle by NASA. The three-sigma level would not include catastrophic events such as shocks.

In the analysis of the tile response, the following additional assumptions were made.

1) The tile was a rectangular, symmetric, rigid mass vibrating perpendicular to the panel only.

2) The SIP was modeled with equivalent linear stiffness and damping elements, to be determined from random tests.

3) The base excitation and acoustic pressure PSD's were initially represented as a constant value over frequencies from zero to a value much greater than the natural frequency of the tile on the SIP. Later analyses removed this restriction.

Base Excitation

For a single degree-of-freedom linear model (Fig. 3), the relationship between input and output is given by[3]

$$S_{\ddot{X}}(\omega) = |H(\omega)|^2 S_{\ddot{y}}(\omega) \tag{1}$$

and the mean square acceleration response is

$$\overline{\ddot{X}^2} = \int_{-\infty}^{\infty} S_{\ddot{X}}(\omega)\,d\omega \tag{2}$$

With the assumption that the panel acceleration PSD is constant over all frequencies, it can be shown[4] that the mean square acceleration of the tile due to base excitation is given by

$$\overline{\ddot{X}^2} = \frac{\bar{S}_{\ddot{y}}\pi f_n(4\zeta^2+1)}{4\zeta} \tag{3}$$

The design stress in the SIP (or at the tile/SIP interface) is given as the three-sigma value. If the mean value of stress is zero, the three-sigma value is

$$P_d = \frac{3W}{A_s}[\overline{\ddot{X}^2}]^{1/2} \tag{4}$$

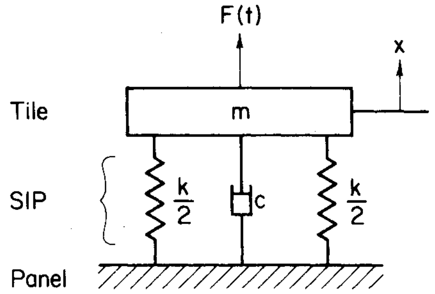

Fig. 4 Forced excitation of a linear mass/spring/damper system.

Finally, substituting Eq. (3) into Eq. (4) yields

$$P_d = \frac{3W}{A_s}\left[\frac{\bar{S}_{\ddot{y}}\pi f_n(4\zeta^2+1)}{4\zeta}\right]^{1/2} \tag{5}$$

Equation (5) is a closed-form expression for the design load in the SIP due to a broadband, random, base excitation. Among other restrictions, it was assumed that the panel acceleration PSD was constant. From a practical viewpoint, if the panel acceleration PSD is constant only in the range of the natural frequency of the tile on the SIP, Eq. (5) will predict a slightly overly conservative value for the load. As shown later in the results, the natural frequency of the tile was usually in the frequency range of the constant-value panel acceleration PSD. Therefore, Eq. (5) provided a simple, conservative expression for base excitation loads.

When the constant-value panel acceleration PSD assumption was relaxed, a numerical solution was derived[4] using the basic expressions given in Eqs. (1) and (2). An expression equivalent to Eq. (3) is

$$\overline{\ddot{X}^2} = \sum_{i=1}^{N} |H(f_i)|^2 S_{\ddot{y}}(f_i)\Delta f_i \tag{6}$$

In Eq. (6) both the transfer function and the panel acceleration PSD are to be evaluated at the center frequency of each one-third octave band. The three-sigma load for the numerical solution is given by substituting Eq. (6) into Eq. (4).

Whether Eq. (3) or Eq. (6) is used to determine the mean square acceleration of the tile on the SIP, it is evident that the dynamic characteristics of the tile/SIP system must be known. In Eq. (3) the tile/SIP natural frequency and damping ratio must be known. In Eq. (6) the tile-panel transfer function must be known for all frequency bands of interest. For a linear single degree-of-freedom oscillator a knowledge of the natural frequency and damping ratio provides a linear transfer function. However, if the transfer function is a nonlinear function of excitation, Eq. (6) can still be used to predict the response. That nonlinear transfer function would have to be determined by random tests under realistic loadings. Since the SIP material was newly developed, detailed dynamic characteristics were not available at the time of this development. Therefore, a linear transfer function (see the Appendix), with equivalent stiffness and damping, was used in predicting the tile loads that are given in the results.

Acoustic Excitation

The dynamic load in the SIP due to the acoustic field, Fig. 2a, was determined in a manner similar to that derived for the base excitation load. The relationship between the acoustic pressure PSD and the tile velocity PSD for a forced, 1 degree-of-freedom system, Fig. 4, is given by[3]

$$S_{\dot{x}}(\omega) = |H_1(\omega)|^2 S_p(\omega) \tag{7}$$

and the mean square velocity response is

$$\overline{\dot{X}^2} = \int_{-\infty}^{\infty} S_{\dot{x}}(\omega)\,\mathrm{d}\omega \tag{8}$$

Assuming that the acoustic pressure PSD is constant, a closed-form solution for the mean square velocity was derived[4] and is given as

$$\overline{\dot{X}^2} = \frac{\bar{S}_p A_t^2}{16\pi\zeta m^2 f_n} \tag{9}$$

In a similar manner, the mean square displacement of the tile due to the acoustic pressure PSD was derived[4]:

$$\overline{X^2} = \frac{\bar{S}_p A_t^2}{64\pi^3\zeta m^2 f_n^3} \tag{10}$$

The three-sigma stresses in the SIP due to the effective damping and stiffness components of the material are

$$P_c = \frac{3c}{A_s}[\overline{\dot{X}^2}]^{1/2} \tag{11}$$

$$P_k = \frac{3k}{A_s}[\overline{X^2}]^{1/2} \tag{12}$$

Since for harmonic motions, damping and stiffness forces are 90 deg out of phase, the total effective stress in the SIP due to the acoustic field is given by

$$P_a = [P_c^2 + P_k^2]^{1/2} \tag{13}$$

Combining Eqs. (9) to (13) yields

$$P_a = \frac{3A_t}{A_s}\left[\frac{\bar{S}_p\pi f_n(4\zeta^2+1)}{4\zeta}\right]^{1/2} \tag{14}$$

As with base excitation, it was assumed initially that the PSD of the acoustic pressure was constant for all frequencies. Like the base excitation PSD, the acoustic pressure PSD may vary by more than an order of magnitude between 20 and 2000 Hz, Fig. 2a. More importantly, the peak value of the acoustic pressure PSD can be at frequencies significantly below the natural frequency of the tile. Thus using the acoustic pressure PSD at the tile natural frequency in Eq. (14) may significantly underestimate the stress in the SIP. A similar statement about the expression for base excitation, Eq. (5), cannot usually be made because the maximum panel acceleration PSD occurred over a wide region of frequencies near the tile natural frequency.

Discretizing Eqs. (7) and (8), an expression for the mean square velocity of the tile is given by

$$\overline{\dot{X}^2} = \sum_{i=1}^{N} |H_1(f_i)|^2 S_p(f_i)\Delta f_i \tag{15}$$

and the mean square displacement is

$$\overline{X^2} = \sum_{i=1}^{N} |H_2(f_i)|^2 S_p(f_i)\Delta f_i \tag{16}$$

where $S_p(f_i)$ is given for the N one-third octave bands by

$$S_p(f_i) = \frac{1}{\Delta f_i}[2.9\times10^{-9}\times10^{(\mathrm{dB}/20)}]^2(\mathrm{psi})^2/\mathrm{Hz} \tag{17}$$

Substitution of Eqs. (15) and (16) into Eqs. (11) and (12) and then into Eq. (13) gives a more accurate value than Eq. (14) for the dynamic stress in the SIP due to the acoustic field.

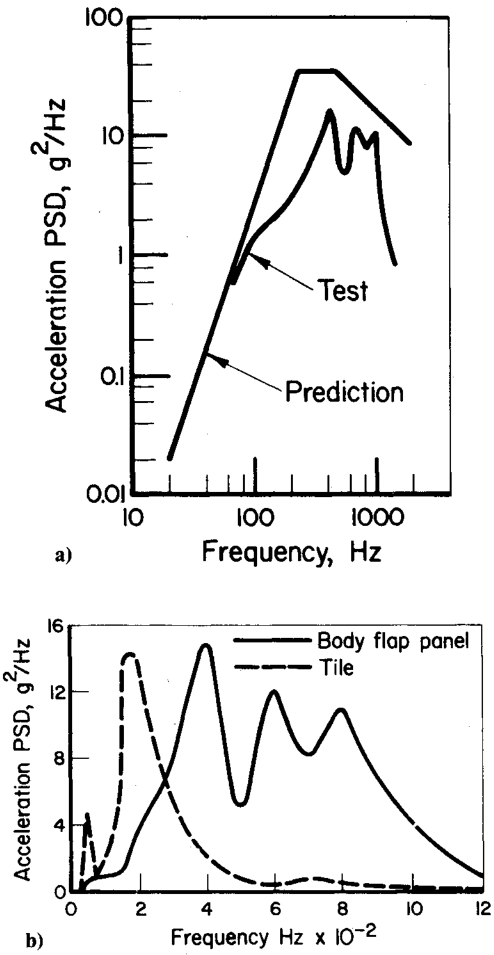

Fig. 5 Vibration results of orbiter body flap test[5]: a) panel vibrations, b) natural frequencies.

The linear transfer functions squared, $|H_1(f_i)|^2$ and $|H_2(f_i)|^2$ are given in the Appendix.

The two dynamic loads given by Eqs. (5) or (14), or their numerical summation equivalents, are two distinct random loading phenomena, but they are not independent. It is therefore assumed that the phasing between the two stresses varies randomly, and that the total three-sigma stress can be expressed as

$$P_t = [P_d^2 + P_a^2]^{1/2} = \frac{3}{A_s}\left[\pi f_n \left(\frac{4\zeta^2+1}{4\zeta}\right)(\bar{S}_p A_t^2 + \bar{S}_{\ddot{y}} W^2)\right]^{1/2} \quad (18)$$

Test Verification

At the time of this modeling effort, very few experimental data were available. But, a full-scale acoustic test of the orbiter's body flap was conducted at NASA JSC during November 1979. The simulated body flap, with mounted tiles, was supported by a scaffold-like structure and subjected to a liftoff acoustic environment of 163 dB O.A., Fig. 2a. During these tests, several panel and tile acceleration PSD's were obtained.[5] An attempt was made to correlate the test results with prediction techniques by using the worst-case panel vibration and the worst-case tile vibration.

In Fig. 5a, the measured acceleration PSD of a body flap panel is compared to the predicted PSD. It can be seen that the test results are well below the prediction envelope. This difference introduces an additional degree of conservatism to the results of the linear random vibration models. Both the maximum panel and tile acceleration PSD responses were plotted on a linear scale to obtain a visual understanding of the vibration, Fig. 5b. Figure 5b shows the tile response with a spectral peak at about 200 Hz.

Table 1 Sensitivity study of three-sigma body flap tile loads—JSC test[a]

Natural frequency, Hz	Damping ratio	Excitation, psi Base	Acoustic	Total
200	0.25	3.9	1.7	4.2
175	0.25	3.1	1.5	3.5
225	0.25	4.6	1.7	4.9
200	0.20	4.2	1.8	4.5
200	0.30	3.8	1.6	4.1

[a]An analysis of test data for the body flap tile (Fig. 5) gave a total three-sigma stress of 4.3 psi (30,000 N/m^2).

The PSD plots of the panel's acceleration and the acoustic field were digitized for analysis at one-third octave bandwidths. The tile mean square acceleration, velocity, and displacement values were calculated for each one-third octave band [Eqs. (6), (15), and (16)]. The transfer functions squared were determined by assuming a tile natural frequency of 200 Hz and a damping ratio of 0.25 (a comparison of panel and tile PSD's, Fig. 5b, established a damping ratio of about 0.25).

Calculations were made to determine the total load, Eq. (18), on the body flap tile due to both base excitation and acoustic excitation; the results are shown in Table 1. Because the tile natural frequency and damping ratio were not well defined, a sensitivity study was conducted by varying these parameters. The parametric changes indicated a maximum variation of less than 17%, compared with the nominal case (row 1).

The measured SIP stress was determined by integrating the tile acceleration PSD from 0 to 1200 Hz, taking the square root, and multiplying by three times the weight divided by the SIP area. This gave a stress of 4.1 psi (28,000 N/m^2) due to the inertia load. The SIP stress due to the acoustic pressure was then calculated by converting the overall SPL to pressure, and then multiplying by three times the tile area divided by the SIP area to give 1.3 psi (9000 N/m^2). Finally, Eq. (18) gave 4.3 psi (30,000 N/m^2).

The excellent correlation between the linear analytical predictions and the single test result provided some confidence that a very complex vibroacoustic problem could be modeled in a relatively simplistic fashion and still produce realistic results.

Predictive Results for Selected Tiles

Four analytical models have been derived to predict loads on the Space Shuttle orbiter's TPS. Two of these models were closed form and computationally easy to apply. The other two models required a summation, but with the aid of a simple computer program, were also straightforward to apply.

To provide a representative survey of tile/SIP loadings due to the vibroacoustic environment, six orbiter tiles were selected for detailed analyses. For each selected tile, drawings were obtained to determine the exact geometry and weight of the tile and SIP, and its location on the orbiter. Using the TPS properties plus the acoustic field[1] and panel vibration specification,[2] all four models were used to predict loads.

When calculating the tile/SIP load due to the panel vibration, the closed-form expression of Eq. (5) was used. The load due to panel vibration using the more exact numerical method was determined by substituting the results of Eq. (6) into Eq. (4). For the acoustic excitation, the closed-form solution was given in Eq. (14). The more exact numerical method for the acoustic excitation results from combining Eqs. (15), (16), (11), (12), and (13). Whether the closed-form or numerical technique was used, the total vibroacoustic stress due to tile random vibrations was found from Eq. (18).

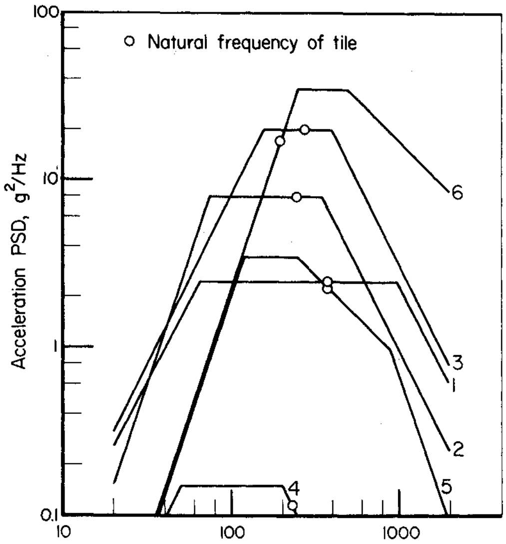

Fig. 6 Orbiter panel vibrations.[2]

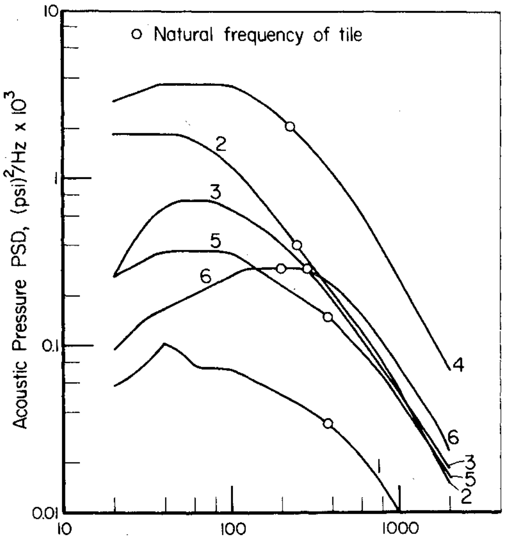

Fig. 7 Orbiter acoustic field.[1]

Table 2 Vibroacoustic analyses parameters

Tile	Orbiter location	Tile natural frequency, Hz	SIP damping ratio	SIP area, in.^{2}	Tile area, in.^{2}	Tile weight, lb
1	Upper wing near tip	380	0.25	21.0	32.0	0.15
2	Rear of front strut	250	0.25	10.0	18.0	0.16
3	Base heat shield	280	0.25	10.0	18.0	0.13
4	Front of front strut	230	0.25	17.6	27.2	0.63
5	Top of vertical fin	380	0.25	9.0	16.0	0.063
6	Center rear body flap	200	0.25	25.0	36.0	0.63

To fully understand the vibroacoustic environment to which the tiles are exposed, the acceleration and acoustic pressure PSD's for six regions of the orbiter are illustrated in Figs. 6 and 7, respectively. These regions were chosen because they were representative of the more severe locations on the orbiter. In Fig. 6 it can be seen that the natural frequency of each tile is either at or near the maximum acceleration PSD frequency range of the particular panel to which it is attached.

On the other hand, the maximum value of acoustic pressure PSD is typically much greater than the acoustic pressure PSD at the natural frequency of the tile, Fig. 7—tile 6 is a notable exception. The obvious implication of this occurrence is that, since the transfer function is near 1.0 for exciting frequency significantly below the natural frequency of the system, most tile/SIP combinations will not greatly amplify the maximum values of the input acoustic forcing.

The natural frequency of a particular tile was scaled by assuming that the $6\times6\times3.5$ in. ($150\times150\times89$ mm) body flap tile had a natural frequency of 200 Hz, Fig. 5. The scaling included the effect of tile weight and the SIP area and thickness. The damping ratio was held constant at 0.25 for all tiles. The weight and exposed surface area were determined from the dimensions and density of the tile. The SIP area was determined by subtracting 0.5 in. (13 mm) off each edge of the tile inner surface and calculating the remaining area. The tile/SIP parameters and orbiter locations for the six tiles analyzed are given in Table 2.

The comparisons between the closed-form and numerical solutions for the base-excited load are given in Table 3. In each case the numerical solution was less than the closed-form solution because the numerical solution did not assume that the panel acceleration PSD was constant. However, it can be seen that when the natural frequency of the tile/SIP occurred within the frequencies of maximum panel acceleration PSD (tiles 1-3 in Fig. 6), the closed-form solution gave only slightly more conservative results than the more accurate numerical solution. When the tile natural frequency did not correspond to the frequencies of maximum panel acceleration (tiles 4-6), the closed-form solution tended to be overly conservative.

Table 3 Prediction of three-sigma, base-excited stress in tile/SIP

Tile	Closed-form solution, psi	Numerical solution, psi
1	1.32	1.29
2	4.33	4.07
3	5.65	5.16
4	1.24	1.06
5	1.52	1.23
6	12.61	8.51

The comparison between the closed-form and numerical solutions for the acoustically excited system are given in Table 4. As for the base-excited load, the numerical solution was always less than the closed-form solution because the acoustic pressure PSD was not constant, Fig. 7. For tile 6, where the tile natural frequency occurred at the same frequency as the maximum acoustic pressure PSD, the closed-form solution was only slightly more conservative than the numerical solution. For tiles 1-5 the closed-form solution was overly conservative compared to the numerical solution, because the tiles greatest response (at its natural frequency) was not at the frequency of greatest acoustic pressure PSD.

Table 4 Prediction of three-sigma, acoustic-excited stress in tile/SIP

Tile	Closed-form solution, psi	Numerical solution, psi
1	1.79	1.10
2	7.28	3.99
3	4.84	3.30
4	8.44	6.58
5	3.95	2.70
6	2.06	1.96

Table 5 Prediction of total three-sigma, stress in tile/SIP due to vibroacoustic levels

Tile	Stress, psi
1	1.7
2	5.7
3	6.1
4	6.7
5	3.0
6	8.7

Table 5 is a summation of the tile/SIP stresses due to the base and acoustic excitation, using the numerical solution technique and Eq. (18). It is of interest to point out that the linear random vibration techniques developed in this paper gave a three-sigma stress of 8.7 psi (60,000 N/m^2) for the body flap tile (tile 6 in Table 5), while the test results gave a three-sigma stress of 4.3 psi (30,000 N/m^2), Table 1. The difference is in the conservatism of the estimated acoustic and vibration environments. For the body flap tile, the random vibration models gave an effective six-sigma stress compared to test data.

Summary

The primary thermal barrier for the Space Shuttle orbiter is a large number of individual, flexibly mounted, ceramic tiles. Each of these tiles is subjected to a broadband acoustic pressure field during liftoff and ascent, which not only loads the tiles, but also causes the orbiter panels to vibrate. In turn, the vibrating panels load the tiles through base excitation.

In this paper, random vibration techniques were used to derive first-order expressions to predict vibroacoustic loads in the SIP caused by broadband random excitations. Two closed-form expressions can be easily applied to provide conservative predictions of SIP stress. In addition, two numerical techniques can be used to provide more accurate predictions.

A comparison of the SIP stress predicted by the numerical techniques with results of a test conducted at NASA JSC correlated well.

To illustrate the analytical techniques, the vibroacoustic loads on six tiles were predicted. In particular, comparisons were made between the closed-form solutions and the more accurate numerical solutions. It was shown that the closed-form methods were only slightly more conservative than the numerical methods when the natural frequency of the tile was within the frequency range of the maximum PSD level of the forcing function.

In spite of the good correlation between the analytical and test results of the orbiter body flap, these methods considered only the bounce degree of freedom of the tile. Further work is needed to determine the stress in the SIP under random loading due to lateral, torsional, and especially, rocking motion of the tile. The rocking motion will be induced on overhung tiles, i.e., tile whose SIP is not symmetrically bonded to the tile. Large rocking motions are likely to cause higher stresses at the edges of the SIP that may result in the tile peeling off its panel.

Appendix

Linear Transfer Functions

For a linear, single degree-of-freedom, base-excited system, Fig. 3, the equation of motion is

$$m\ddot{x}+c\dot{x}+kx=c\dot{y}+ky \tag{A1}$$

Assuming harmonic motion, the linear transfer function squared between the acceleration of the mass and the acceleration of the base is

$$|H(f_i)|^2=|H(\omega)|^2=\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2} \tag{A2}$$

For a linear, single degree-of-freedom, forced system, Fig. 4, the equation of motion is

$$m\ddot{x}+c\dot{x}+kx=F(t) \tag{A3}$$

Assuming harmonic motion, the linear transfer function squared between the velocity of the mass and the pressure acting on the surface is

$$|H_1(\omega)|^2=|H_1(f_i)|^2=\frac{(2\pi f_i A_t/k)^2}{(1-r^2)^2+(2\zeta r)^2} \tag{A4}$$

The linear transfer function squared between the displacement of the mass and the pressure acting on the surface is

$$|H_2(\omega)|^2=|H_2(f_i)|^2=\frac{(A_t/k)^2}{(1-r^2)^2+(2\zeta r)^2} \tag{A5}$$

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